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3 (Sem-4/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY-HC-4016

(Mathematical Physics-III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any seven** questions of the following : $1 \times 7 = 7$

(a) What is the argument of $-3i$?

(b) Express $f(z) = z^2$ in the form of $u(x, y) + iv(x, y)$.

(e) What is singular point of an analytic function?

$$\frac{\partial u}{\partial x} = \frac{1}{x} \cdot \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}$$

(d) Evaluate $\delta_q^p A_s^{qr}$.

(e) State the shifting property of Fourier transform (FT).

(f) Find the residue of the complex function $f(z) = \frac{1}{z^2 + 1}$ at the pole $z = i$.

(g) Show that $L(1) = \frac{1}{s}$, $s > 0$.

(h) What is rank of a tensor? Give one example of a zero rank tensor.

(i) Define Fourier inverse transform.

(j) Write the polar form of a complex number.

2. Answer **any four** of the following questions :

$$2 \times 4 = 8$$

(a) Check whether the function $\log z$ is analytic or not.

(b) Plot the complex number $e^{(1-\pi/6i)}$ in Argand diagram.

$$\frac{\partial h}{\partial x} = \frac{1}{x} \cdot \frac{\partial y}{\partial x}$$

$$\frac{\partial h}{\partial x} \times \frac{\partial x}{\partial y}$$

- (d) Evaluate $\delta_q^p A_s^{qr}$.
- (e) State the shifting property of Fourier transform (FT).
- (f) Find the residue of the complex function $f(z) = \frac{1}{z^2 + 1}$ at the pole $z = i$.
- (g) Show that $L(1) = \frac{1}{s}$, $s > 0$.
- (h) What is rank of a tensor? Give one example of a zero rank tensor.
- (i) Define Fourier inverse transform.
- (j) Write the polar form of a complex number.

2. Answer **any four** of the following questions :
 $2 \times 4 = 8$

- (a) Check whether the function $\log z$ is analytic or not.
- (b) Plot the complex number $e^{(1-\pi/6i)}$ in Argand diagram.

(c) Prove that the contraction of the tensor A_m^l is invariant.

✓ (d) Obtain the Fourier transform of the function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(e) Using the property of Levi-Civita symbol prove that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$.

✓ (f) If $L[f(x)] = \bar{f}(s)$, then show that $L[e^{ax} f(x)] = \bar{f}(s-a)$.

(g) Evaluate the integral $\oint \frac{dz}{z}$ around a unit circle.

✓ (h) Expand the function

$f(z) = \frac{1}{z+1}$, about $z=1$ in Taylor series up to two terms.

3. Answer **any three** questions of the following : 5×3=15

(i) Find the value of the integral

$$\int_0^{1+i} (x - y - ix^2) dz, \text{ along real axis from}$$

$z = 0$ to $z = 1$ and then along the line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

(ii) State and prove Cauchy's integral formula.

(iii) Obtain the Fourier sine and cosine transform of the function

$$f(x) = \begin{cases} 1, & 0 < x < \pi/2 \\ 0, & x > \pi/2 \end{cases}$$

(iv) What is Kronecker delta? Show that it is a mixed tensor of rank 2. 2+3=5

(v) Find the Laplace transform of the function $f(t) = \sin at$.

(vi) Show that $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
and $\text{Arg}(z_1 \cdot z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$.

(vii) What are raising and lowering of indices of a tensor? Prove that the raising and lowering operation of indices are reciprocal to each other. $2+3=5$

(viii) Evaluate $\oint_C \frac{\cos z}{z} dz$, where C is an ellipse given by $9x^2 + 4y^2 = 1$, using Cauchy's integral formula. 5

4. Answer **any three** of the following questions: $10 \times 3 = 30$

(a) (i) Show that if $f(z) = u + iv$ is an analytic function and $\vec{F} = \hat{i}v + \hat{j}u$ is a vector, then $\text{div } \vec{F} = 0$ and $\text{curl } \vec{F} = 0$ are equivalent to Cauchy-Reimann equations. 6

(ii) State and prove quotient law of tensors. 4

(b) (i) The Laplace transform of $\sin 3t$ is $\frac{3}{s^2 + 9}$ and the Laplace

transform of $\cos 5t$ is $\frac{s}{s^2 + 25}$.

Find the Laplace transform of $5 \sin 3t + 3 \cos 5t$ using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of $\frac{4S+5}{(S-1)^2(S+2)}$. 5

(c) (i) If A_λ is a covariant tensor of rank 1, show that $\frac{\partial A_\lambda}{\partial x_\mu}$ is not a tensor. 3

(ii) Prove the following identities: 2+2+3=7

(a) $\delta_{ii} = 3$

(b) $\delta_{ik}\epsilon_{ikm} = 0$

(c) $\epsilon_{iks}\epsilon_{mps} = \delta_{im}\delta_{kp} - \delta_{ip}\delta_{km} = 0$

(d) State and prove Fourier integral theorem.

(e) (i) Using the method of residues, show that $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$. 6

(ii) Express the complex number $1+2i/1-3i$ in $r(\cos\theta+i\sin\theta)$ form. 4

(f) Evaluate **any two** of the following integrals by contour integration :

5×2=10

(i) $\int_0^{\infty} \frac{dx}{x^2+1}$

(ii) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$

(iii) $\int_{-\infty}^{+\infty} \frac{e^{ax}}{1+e^x} dx$

(g) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

under the conditions that, $y(x, 0) = 0$,

$y'(x, 0) = 0, x > 0$ and $y(0, t) = t$,

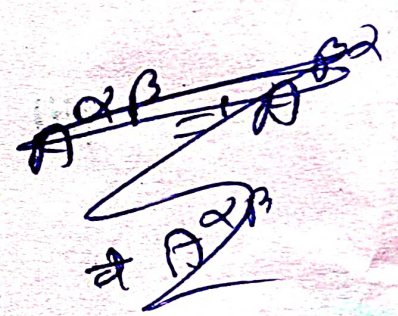
$\lim_{x \rightarrow \infty} y(x, t) = 0, t \geq 0$.

(h) (i) What is residue of a complex function? State and prove Cauchy's residue theorem.

1+1+4=6

(ii) Show that any contravariant or covariant tensor of the second order can be resolved into symmetric and antisymmetric parts.

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(b) Solve the wave equation under the conditions that $y(x, 0) = 0$, $y(x, l) = 0$, $\lim_{x \rightarrow 0} y(x, t) = 0$, $\lim_{x \rightarrow l} y(x, t) = 0$. What is residue of a complex function? State and prove Cauchy's residue theorem.