Total number of printed pages–7

3 (Sem–1/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics–I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : 1×7=7

   (a) Define unit vectors.

   (b) If $\vec{A}.\vec{B} = 0$, then what is the angle between $\vec{A}$ and $\vec{B}$?

   (c) What is a 'DEL' operator ?

   (d) Find the Laplacian of the scalar field $\phi = xy^2z^3$

Contd.

(e) State Green's theorem.

(f) Write the order and degree of the differential equation

$$2y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^4=0$$

(g) What do you understand by the statement $\vec{\nabla}.\vec{A}=0$?

(h) What is an 'error' in statistics ?

(i) Define coordinate surfaces in curvilinear co-ordinates.

(j) Write the integrating factor of the differential equation

$$\frac{dy}{dx}+5y=x^2$$

(k) Write the geometrical interpretation of the scalar triple product.

(l) Define variance in statistics.

2. Answer ***any four*** of the following questions : 2×4=8

(a) Give examples of a scalar field and a vector field.

(b) If $\vec{r}$ represents the position vector, then find the value of $\vec{\nabla}.\vec{r}$.

(c) Define the line integral of a vector.

(d) Write down the relation of cylindrical co-ordinate (r,θ,z) with cartesian co-ordinate (x,y,z).

(e) Explain the scale factors h_1, h_2, h_3 in curvilinear co-ordinate system.

(f) For what value of N, the vectors $\vec{A}=2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{B}=N\hat{i}+2\hat{j}+2\hat{k}$ are perpendicular to each other.

(g) Evaluate $\iint_S \vec{r}.\hat{n}\,ds$, where S is a closed surface.

(h) Prove that $\delta(x)=\delta(-x)$.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Show that

$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$

(b) If $\phi = xy + yz + zx$ and $\vec{F} = \vec{\nabla}\phi$, then find $\vec{\nabla}.\vec{F}$ and $\vec{\nabla}\times\vec{F}$.

(c) Apply Green's theorem in the plane to evaluate the integral

$$\oint_C \left[\left(xy - x^2\right)dx + x^2 y\, dy\right]$$

over the triangle bounded by the lines $y = 0$, $x = 1$ and $y = x$.

(d) Solve the differential equation

$$2xy\frac{dy}{dx} = x^2 + 3y^2$$

(e) Express $\nabla^2\psi$ in cylindrical coordinate system.

(f) Prove that

$$\delta\left(x^2 - a^2\right) = \frac{1}{2a}\left[\delta(x-a) + \delta(x+a)\right]$$

(g) A function is defined as

$$f(x) = \begin{cases} 0 \text{ for } x < 2 \\ \frac{1}{18}(2x+3) \text{ for } 2 \le x \le 4 \\ 0 \text{ for } x > 2 \end{cases}$$

Show that it is a probability density function.

(h) If $\vec{F}$ is a vector, prove that

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{F}\right) - \nabla^2\vec{F}$$

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Show that the gradient of a scalar field is a vector. 5

(ii) Show that 2½×2=5

1. $div\ curl\ \vec{A} = 0$ and

2. $curl(grad\,\phi) = 0$

(b) (i) Define curvilinear co-ordinate system. When it is called orthogonal ? 3+1=4

(ii) Obtain expression for length, area and volume elements in curvilinear coordinate system. 2+2+2=6

(c) (i) State and explain Gauss-divergence theorem. 3

(ii) Give the physical meaning of divergence and curl of a vector. 2+2=4

(iii) Find an expression of $\vec{\nabla}.\vec{A}$ in spherical polar co-ordinate system. 3

(d) (i) Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 5

(ii) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$. 5

(e) Solve the following differential equations : 5+5=10

(i) $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

(ii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

(f) State and prove Stoke's theorem. Using Stoke's theorem show that

$\oint_C \vec{r} \times d\vec{r} = 2\iint_S d\vec{S}$, where C is the closed perimeter curve bounding the open surface S. 1+5+4=10

(g) (i) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, subject to the condition $y(0) = 0, y'(0) = 1$ 6

(ii) Prove that

$\vec{A}.(\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}).\vec{C}$ 4

(h) (i) If $\vec{A} = 6\hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j} - 3\hat{k}$

$\vec{C} = \hat{i} + \hat{j} + \hat{k}$ then evaluate

$\vec{A} \times (\vec{B} \times \vec{C})$ 4

(ii) Evaluate $\oint_C x^2y\,dx + y^2dy$, where C is the boundary of the region enclosed by $y = x$ and $y^2 = x$. 6
