

Sessional Exam-2023

Paper Name: Mathematical Physics III  
Paper Code: PHY-HC-4016

Total marks: 30

Time: 1.5 hrs.

Section A

(2 × 1 = 2)

1. Answer any two (02)

- Define simply and multiply connected region.
- What does the equation  $|z| = 1$  represent?
- What is the singular point of an analytic function?
- Find the type of singularity of  $f(z) = \frac{\sin z}{z^2}$ .

(1 × 3 = 3)

2. Answer any one (01)

- Find the residue of  $f(z) = \frac{e^{iz}}{z^2 + a^2}$  at  $z = -ia$ .
- Evaluate  $\oint_C \frac{(\sin \pi z^2 + \cos \pi z^2)}{(z-1)(z-3)} dz$ , where C is the circle where  $|z| = 2$ .
- Evaluate  $\oint_C \frac{e^z}{(z+1)^3} dz$ , where C is the circle where  $|z| = 2$ .

(2 × 5 = 10)

3. Answer any two (02)

- Prove Cauchy's integral formula if the function  $f(z)$  is analytic inside and on the boundary C of a simply-connected region R.
- State and prove Cauchy's residue theorem.
- State the Taylor Theorem in complex analysis. Expand  $f(z) = \ln(1+z)$  in Taylor series about  $z = 0$ .
- State the Cauchy Reimann conditions for an analytic function. Show that the function  $\ln(z)$  is analytic and hence find the derivative  $f'(z)$ .

Section B

(2 × 1 = 2)

1. Answer any two (02)

- In which coordinate system there is no difference between covariant and contravariant tensor.
- Which of the following symbols does not represent tensor?  
(i)  $A_j^i$       (ii)  $A^{ii}$       (iii)  $A_i^j$       (iv)  $A_k^{ij}$
- What is the value of  $\delta_j^i$  in 3-dimensional space?
- What will be the number of components of a tensor of rank  $r$  in a space of dimension  $N$ ?

(1 × 3 = 3)

2. Answer any one (01)

- Show that Kronecker delta is a tensor of rank two.
- What do you mean by contraction in a tensor analysis? If the tensor  $A_k^{ij}$  is contracted once what will be the rank of the new tensor?
- Define outer multiplication of tensors. What will be the rank of outer multiplication of the tensors  $A_k^{ij}$  and  $B_{qr}^p$ ?

(2 × 5 = 10)

3. Answer any two (02)

- Define contravariant and covariant vector. Show that addition of two tensors is also a tensor.
- Define symmetric and antisymmetric tensor. Show that a tensor of rank 2 can be expressed as the sum of a symmetric and an antisymmetric tensor. Is the symmetric property of a tensor independent of co-ordinate system?
- State and prove Quotient law.
- The velocity components in cartesian coordinate system are  $\dot{x}$  and  $\dot{y}$ . Find the velocity components in plane polar coordinate.