

Total number of printed pages-7

1 (Sem-4) PHY 2

2025

**PHYSICS**

Paper : PHY0400204

**( Quantum Mechanics )**

Full Marks : 45

Time : 2 hours

**The figures in the margin indicate full marks for the questions.**

*Symbols have their usual meanings.*

1. Objective-type : (Answer **all** questions)

1×5=5

(a) If  $\lambda_c$  is the Compton shift, what is the greatest wavelength change in Compton scattering ?

(b) What is the outcome of  $[\hat{x}, e^{\hat{x}}]$  ?

(c) Plot the wavefunction  $\psi(x) = \frac{1}{a^2} x e^{-x/a}$  for  $x > 0$ , where  $a$  is a constant and real number.

(d) What is the requirement of de Broglie wavelength of electron for the diffraction of electrons by a crystal?

(e) What is the total degeneracy in energy of H-atom with principal quantum number  $n = 2$ ?

2. Very short answer-type : (Answer any five questions)  $2 \times 5 = 10$

(a) Write Planck's blackbody radiation formula and obtain Rayleigh-Jeans formula under limiting condition.

(b) Write the general form of the eigenvalue equations for the Hamiltonian of a one-dimensional linear harmonic oscillator and mention possible eigenvalues.

(c) What is the minimum value of the product  $\Delta x \Delta p_x$ ? Plot  $\Delta p_x$  versus  $\Delta x$ .  $1+1=2$

B06FN 0151

2

(d) Why group velocity and not the phase velocity is considered to describe the velocity of a moving material particle?

(e) What is zero-point energy? Why it cannot be equal to zero for a particle confined in a potential box?  $1+1=2$

(f) If  $\lambda_p$  and  $\lambda_\alpha$  are the de Broglie wavelengths of a proton and an alpha particle moving with same non-relativistic speeds, then find the ratio

$$\frac{\lambda_p}{\lambda_\alpha}$$

(g) Write the differential forms of linear momentum and energy operators. Is momentum operator Hermitian?  $1+1=2$

(h) An attempt is made to measure the position of an electron in an atom. The uncertainty of this measurement is  $1\text{\AA}$ . What is the minimum uncertainty in the measurement of linear momentum of the electron?

B06FN 0151

3

Contd.

(i) Using the general expression for spherical harmonics  $Y_l^{ml}(\theta, \varphi)$ , evaluate  $Y_1^0$ .

(j) Write the eigenvalue equations for the z-component of orbital angular momentum and square of the orbital angular momentum operators for a particle under the action of a spherically symmetric potential.

3. Short answer-type : (Answer **any four** questions) 5×4=20

(a) Obtain the normalization constant by normalizing the given wavefunction

$$\psi(x) = \frac{3}{\sqrt{10}}(a^2 - x^2) \text{ in the region}$$

$-a \leq x \leq a$ . Hence, show the variation of the normalized wavefunction with  $x$  graphically with a mention of the peak value. 3+2=5

(b) An experiment on photoelectric effect is conducted for a metal. The stopping potentials are 4.50V and 0.20V corresponding to light wavelengths 190nm and 550nm, respectively. Find the work function of the metal.

(c) Provide a brief physical interpretation of wavefunction.

(d) Consider a beam of particles of mass  $m$ , moving in the positive  $x$  direction with energy  $E$  towards a potential step at  $x = 0$ . The potential  $V(x)$  is zero for  $x \leq 0$  and it is  $\frac{3}{4}E$  for  $x > 0$ . Find the reflection coefficient.

(e) Starting with time dependent Schrödinger equation, obtain the differential form of equation of continuity involving the probability current density.

(f) Briefly describe the Davisson-Germer experiment that confirms wave nature of electrons.

(g) Write the time-independent Schrödinger equation for a one-dimensional linear harmonic oscillator and provide its ground state solution using the Hermite polynomials.

(h) Find the outcome of the commutation relation,

$$[\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{y}\hat{p}_z - \hat{z}\hat{p}_y]$$

4. Essay-type : (Answer **any one** question)

10×1=10

(a) Starting with the concept of wave packet, obtain the intensity distribution. Introduce the Gaussian form of wave packet and briefly explain its connection with a moving material particle.  
7+3=10

(b) Write the time independent Schrödinger equation in three dimensions for a particle experiencing central potential and obtain its radial and angular parts in spherical polar coordinate system. Using the idea of separation of variables, find the normalized azimuthal wavefunction.  
7+3=10

(c) Consider a particle inside a one-dimensional potential box having infinite potential barriers at  $x=0$  and  $x=L$ . The wavefunction of the particle is  $\psi(x) = Nx(L-x)$ , where  $N$  is the normalization constant. Find the expectation values of position and linear momentum operators.

\_\_\_\_\_