

**Four Year Undergraduate Course in Physics
Semester - IV**

Paper Name: Mathematical Physics

Paper Code= PHY040404

Total number of lectures= 45

**Total credits = 4 (Theory 3 +Laboratory 1) (Total Marks 100: Internal-
25+External-75)**

Course Objective:

- To solve partial differential equations using separation of variables, including Laplace's equation and the wave equation.
- To apply Fourier series expansion to represent periodic functions using sine and cosine functions.
- To understand complex analysis principles, including analytic functions, integration and residue theorem.
- To develop proficiency in tensor algebra, covering transformations, contravariant and covariant tensors and tensor algebra.
- To gain a preliminary knowledge to probability theory, focusing on independent random variables, probability distributions, and mean and variance calculations.

Course outcome: On successful completion of the course, the students will be equipped with the techniques related to solving partial differential equations using separation of variables method, application of Fourier series analysis, solving complex integrations, dealing with tensors and probability distributions which are relevant while dealing with wave mechanics, electrodynamics, quantum mechanics, theory of relativity and experimental physics.

Theory

Unit I: Partial Differential Equations

(Lectures 10)

Solutions to partial differential equations, using separation of variables: Laplace's Equation in problems of rectangular, cylindrical and spherical symmetry. Wave equation and its solution for vibrational modes of a stretched string, rectangular and circular membranes.

Unit II: Fourier Series

(Lectures 07)

Periodic functions. Orthogonality of sine and cosine functions, Dirichlet Conditions (Statement only). Expansion of periodic functions in a series of sine and cosine functions and determination of Fourier coefficients. Complex representation of Fourier series. Application to square and triangular waves.

Unit III: Complex Analysis

(Lectures 17)

Functions of Complex Variables. Analyticity and Cauchy-Riemann Conditions. Examples of analytic functions. Singular functions: poles and branch points, order of singularity. Integration of functions with complex variable. Cauchy's Integral theorem and Cauchy's Integral formula. Simply and multiply connected regions. Laurent and Taylor's series expansions. Residue Theorem with application.

Unit IV: Tensor Algebra

(Lectures 06)

Introduction to tensor, Transformation of co-ordinates, Einsteins summation convention. Contravariant, covariant and mixed tensors. Symmetric and antisymmetric tensors, Kronecker delta, LeviCivita tensor. Quotient law of tensors. Rules of combination of tensors: addition, subtraction, outer multiplication, contraction and inner multiplication.

Unit V: Introduction to Probability

(Lectures 05)

Independent random variables: Probability distribution functions; binomial, Gaussian and Poisson, with examples. Mean and variance.

References

1. Mathematical Physics; H K Dass and R Verma, S Chand and Company limited.
2. Mathematical methods for Physics and Engineering; K. F Riley, M. P Hobson, S.J Bence, Cambridge University Press.
3. Graduate Mathematical Physics (With Mathematica Supplement); J J Kelly, Willey-VCH Verlag GmbH and Co. KGaA.
4. Mathematical Methods for Physicists; G. B. Arfken, H. J. Weber and F.E. Harris, Elsevier.
5. Ordinary and Partial Differential equations; M. D Raisinghania, S. Chand and Company Ltd.
6. Complex Variables; M R Spiegel, S Lipschutz, J J Schiller and D Spellman, Schaum's Outline Series, McGraw Hill Education.
7. Complex variables Demystified (A self teaching guide); D McMahan, McGraw Hill Education.
8. A Student's Guide to vectors and Tensors; D A Fleisch, Cambridge University Press.
9. Vector analysis and an introduction to Tensor analysis; S Lipschutz, D Spellman, M R Spiegel, Schaum's Outline Series, McGraw Hill Education.
10. Tensors and applications with Scilab Programs; N D Soni, I.K International Publishing House Pvt. Limited.
11. Probability and Statistics; M R Spiegel, J J Schiller and R A Srinivasan, Schaum's Outline Series, McGraw Hill Education.

Laboratory

(Use C/C++/Scilab/FORTRAN/Mathematica/ Matlab/ Python to solve the following problems.)

1. Solve the differential equations

$$\frac{dy}{dx} = e^x \text{ with } y = 0 \text{ for } x = 0$$

$$\frac{dy}{dx} + e^{-x}y = x^2$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -y$$

$$\frac{d^2y}{dx^2} + e^{-x}\frac{dy}{dx} = -y$$

2. Perform the multiplication of two 3×3 matrices.
3. Compute the eigenvalues and eigenvectors of the following matrices.

$$\begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & -i & 3+4i \\ i & 2 & 4 \\ 3-4i & 4 & 3 \end{bmatrix}, \begin{bmatrix} 2 & -i & 2i \\ i & 4 & 3 \\ -2i & 3 & 5 \end{bmatrix}$$

4. Using random number compute the areas of circle, square, volume of sphere and value of pi (π).
5. Evaluate trigonometric functions e.g. $\sin\theta$; $\cos\theta$; $\tan\theta$ etc. using Interpolation by Newton Gregory Forward and Backward difference formula.
6. Find the solution of Partial Differential Equations: (a) Wave equation (b) Heat equation.
7. Evaluate the integral I , where,

$$I = \frac{1}{\sqrt{2\pi\sigma^2}} \int \exp\left[-\frac{(x-2)^2}{2\sigma^2}\right] (x+3) dx \text{ for } \sigma = 1.0, 0.1, 0.01 \text{ and show that } I \rightarrow 5$$

8. Compute the n th roots of unity for $n = 2, 3$, and 4.
9. Find the two square roots of $5 + 12i$.